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An analysis of stability and oscillation modes in boiling multichannel loops using parameter perturbation methods

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Abstract—Systems consisting of parallel boiling channels coupled with an external loop are studied. The theory of perturbation of parameter-dependent linear operators is applied. This method permits the analysis of asymmetric systems. Simple analytical criteria are found which greatly reduce the complexity of the analysis required for coupled parallel channels. In addition, it is possible with this analysis to explain many of the seemingly contradictory results of prior experiments.

INTRODUCTION

MANY experimental and analytical studies have been performed in order to understand instability mechanisms in boiling channels. As a consequence, the density-wave instability phenomenon is rather well-understood in single channels and in boiling loops. In contrast, our understanding of instability mechanisms in coupled parallel channels is not well developed. Different experiments carried out in multichannel systems have often yielded seemingly contradictory results [1–6]. In particular, different results have been reported concerning the modes of oscillation.

Gerliga and Dulevski [7] derived the characteristic equation for multichannel stability. Fukuda and Htasegawa [8] analyzed oscillation modes using a matrix technique. A general procedure for the stability analysis of multichannel coupled systems, such as boiling water nuclear reactors, was developed in ref. [9]. Taleyarkhan *et al.* [10] developed a theory to analyze ventilated parallel channels. In refs. [1, 11, 12] parallel channel-to-channel oscillations were analyzed in the time domain. Guido *et al.* [13] analyzed the stability of two parallel channels using a parameter perturbation technique.

It is the purpose of this paper to quantify the instability modes which may occur when boiling parallel channels interact with each other and with the hydraulic loop in which they are installed. A symmetric system of identical parallel channels was taken as a reference case. The theory of parameter perturbation was applied to generalize the results to asymmetric cases.

ANALYSIS

Let us consider a system consisting of a set of boiling parallel channels, having common inlet and outlet plena, which are coupled with an external loop. Such a system is shown schematically in Fig. 1. Let us assume the validity of either a one-dimensional homogeneous equilibrium or drift-flux model, applied to a system operating at constant pressure. Using the equation of state, variations in the fluid density can be incorporated as variations in the fluid enthalpy. Choosing as state variables the enthalpy, pressure and flow rate, we have

$$h(z,t) = h_0(z)+\Delta h(z,t) \tag{1a}$$

$$p(z,t) = p_0(z)+\Delta p(z,t) \tag{1b}$$

$$w(z,t) = w_0(z)+\Delta w(z,t) \tag{1c}$$

where h_0, p_0 and w_0 are the local steady-state enthalpy, pressure, and flow rate, respectively, and Δh, Δp and Δw are the corresponding perturbations. Equations (1) can be used to linearize the conservation equations. The solutions of the linearized equations can be written as

$$\Delta h(z,t) = \delta h(z,s)\,e^{st} \tag{2a}$$

$$\Delta p(z,t) = \delta p(z,s)\,e^{st} \tag{2b}$$

$$\Delta w(z,t) = \delta w(z,s)\,e^{st} \tag{2c}$$

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NOMENCLATURE

A cross-sectional area
C transfer function defined in equation (12)
E transfer function defined in equation (8)
f friction coefficient
G transfer function defined in equation (3)
g gravity
H transfer function defined in equation (6b)
h enthalpy
δh function defined in equation (2a)
Δh perturbation of the enthalpy about the steady state
I transfer function defined in equation (6a)
J transfer function defined in equation (3)
k control parameter characterizing each channel
$\bar{k}$ average value of the control parameter
Δk_i parameter perturbation defined in equation (23)
N number of channels
Δp pressure drop
q' power per unit axial length
s solution of the characteristic equation
Δs perturbation of the solution of the characteristic equation, defined in equation (24)
t time
u fluid velocity
v specific volume
w flow
δw function defined in equation (2b)
Δw perturbation of the flow about the steady state
z axial position.

Greek symbols
λ boiling boundary
ρ density
σ real part of the solution of the characteristic equation
ω imaginary part of the solution of the characteristic equation
Ω parameter defined in equation (A8).

Subscripts
C boiling channel
e exit
E two-phase part of the external loop
f liquid phase
I single-phase part of the external loop
i channel i; inlet
j channel j
N Nth channel
n nth mode
st stable channel
un unstable channel
0 steady state
1 first channel
2 second channel
ω evaluated for $s = i\omega$.

Superscripts
(n) nth mode
(1) first mode
(2) second mode
0 system with identical channels.

where s is, in general, a complex number. Substituting equations (2) into the linearized conservation equations of mass, energy, and momentum [14] and integrating in space, the pressure drop perturbation in each component of the loop can be related to the flow and enthalpy perturbations at the inlet of each component. The resultant expressions are similar to those obtained by Laplace transforming the linearized conservation equations. For example, for any component j

$$\delta\Delta p_j(s) = G_j(s)\delta w_j(s) + J_j(s)\delta h_j(s) \qquad (3)$$

where δw_j and δh_j are the values of $\delta h(z,s)$ and $\delta w(z,s)$ evaluated at the inlet of component j of the loop. Expressions for $G_j(s)$ and $J_j(s)$ for a variety of components are available in the literature [14, 15]. Moreover, the Appendix presents a derivation of these transfer functions for a boiling channel.

In this study, constant enthalpy is assumed in the adiabatic single-phase part of the loop. Then, pressure drop perturbations in the system shown in Fig. 1 can be written as

$$\delta\Delta p_{\mathrm{I}} = G_{\mathrm{I}}(s) \sum_{i=1}^{N} \delta w_i \qquad (4a)$$

$$\delta\Delta p_{\mathrm{E}} = G_{\mathrm{E}}(s)\delta w_{\mathrm{E}} + J_{\mathrm{E}}(s)\delta h_{\mathrm{E}} \qquad (4b)$$

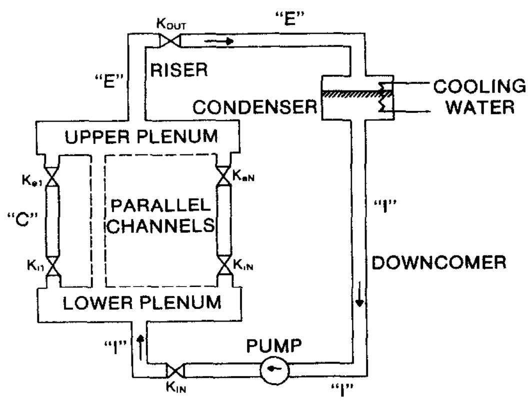

FIG. 1. A loop containing parallel boiling channels.

$$\delta\Delta p_{\mathrm{C}} = G_i(s)\delta w_i \quad (i = 1,2,\ldots,N) \tag{4c}$$

where w_{E} is the summation of all channel exit flows, and the average enthalpy, h_{E}, is defined by

$$h_{\mathrm{E}} = \frac{\sum_{i=1}^{N} h_{\mathrm{E},i} w_{\mathrm{E},i}}{w_{\mathrm{E}}}. \tag{5}$$

Perturbing and integrating (in space) the equations of mass and energy conservation, the enthalpy and flow perturbations at the exit of a boiling channel can be related to the flow perturbations at the inlet by

$$\delta w_{\mathrm{E},i} = I_i(s)\delta w_i \tag{6a}$$

$$\delta h_{\mathrm{E},i} = H_i(s)\delta w_i. \tag{6b}$$

Expressions for the transfer functions $I_i(s)$ and $H_i(s)$ can also be found in the literature [14, 15].

Perturbing equation (5) and using equation (6) and (4b) yields

$$\delta\Delta p_{\mathrm{E}} = \sum_{i=1}^{N} E_i(s)\delta w_i \tag{7}$$

where

$$E_i(s) \triangleq G_{\mathrm{E}}(s) + \frac{J_{\mathrm{E}}(s)}{w_{\mathrm{E},0}}\left[w_{\mathrm{E}i,0}H_i(s) + (h_{\mathrm{E}i,0} - h_{\mathrm{E},0})I_i(s)\right] \tag{8}$$

and subscript '0' refers to the steady-state condition.

The characteristic equation

The pressure drops satisfy the Kirchhoff law along the hydraulic loop

$$\delta\Delta p_{\mathrm{I}} + \delta\Delta p_{\mathrm{C}} + \delta\Delta p_{\mathrm{E}} = 0. \tag{9}$$

Since the parallel channels have the same imposed pressure drop, equation (4c) may be used to obtain

$$\delta\Delta p_{C} = \sum_{i=1}^{N} \frac{G_i(s)}{N}\delta w_i. \tag{10}$$

Combining equations (4a), (7) and (10) with equation (9) yields

$$\sum_{i=1}^{N} C_i(s)\delta w_i = 0 \tag{11}$$

where

$$C_i(s) \triangleq G_{\mathrm{I}}(s) + \frac{G_i(s)}{N} + E_i(s). \tag{12}$$

Due to the fact that the pressure drops are the same for all parallel channels, equation (4c) yields

$$G_j(s)\delta w_j = G_i(s)\delta w_i. \tag{13}$$

Equations (11) and (13) comprise a system of N homogeneous equations with the N unknowns, δw_i. In matrix form this system of equations can be written as

$$\begin{bmatrix} C_1 & C_2 & C_3 & C_4 & \ldots & C_N \\ G_1 & -G_2 & 0 & 0 & \ldots & 0 \\ 0 & G_2 & -G_3 & 0 & \ldots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & G_{N-1} & -G_N \end{bmatrix} \cdot \begin{bmatrix} \delta w_1 \\ \delta w_2 \\ \vdots \\ \delta w_N \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \vdots \\ 0 \end{bmatrix} \tag{14a}$$

or

$$\mathbf{M}\,\delta\mathbf{w} = 0. \tag{14b}$$

In order for equations (14) to have a non-zero solution for $\delta\mathbf{w} = (\delta w_1, \delta w_2, \ldots, \delta w_N)^{\mathrm{T}}$, it is necessary and sufficient that the determinant of matrix $\mathbf{M}$ be zero. This, in turn, yields the following characteristic equation of the coupled multichannel system and the associated external loop

$$\sum_{i=1}^{N} C_i(s) \prod_{j \neq i} G_j(s) = 0. \tag{15}$$

The roots of equation (15) are in general complex (i.e. $s_n = \sigma_n + \mathrm{i}\omega_n$). Stability of the system requires that all roots have negative real parts (i.e. $\sigma_n < 0$). Corresponding to each solution, s_n, of equation (15), there is a phasor $\mathbf{w}^{(n)}$ which satisfies equations (14). However, since equation (14a) is a homogeneous system, an arbitrary constant remains undetermined.

The general solution of the problem of N-parallel channels coupled with an external loop is given by a linear combination of functions of the form given by equations (2), where s is replaced by the solutions, s_n, of the characteristic equation. The initial conditions, $h(z, 0)$ and $w_i(0)$, for each channel are needed to determine the coefficients in the general solution.

At the neutral stability threshold, there are two conjugate roots of the characteristic equation with null real parts (i.e. $s = \pm\mathrm{i}\omega$). All other roots must have negative real parts, and thus the exponentials in equations (2) for these roots tend to zero as time increases. Then, the behavior of the system for large times is given by equations (2) evaluated at $s = \pm\mathrm{i}\omega$. The asymptotic response of the inlet flow of each channel is thus given by

$$\delta\mathbf{w}(t) = A\,Re\,(\mathbf{w}_{\omega}\,\mathrm{e}^{\mathrm{i}\omega t}) \tag{16}$$

where A is determined by the initial conditions. The phasor, $\mathbf{w}_{\omega}$, is obtained by solving the equation

$$\mathbf{M}(\mathrm{i}\omega)\mathbf{w}_{\omega} = 0. \tag{17}$$

Systems having identical channels

If all the parallel boiling channels in the system are identical (i.e. $G_i(s) = G(s)$ and $C_i(s) = C(s)$, for all i), then equation (15) reduces to

$$C(s)G^{N-1}(s) = 0. \quad (18)$$

Equation (18) yields two sets of solutions. The first set satisfies the equation

$$C(s) \triangleq G_1(s) + \frac{1}{N}G(s) + E(s) = 0. \quad (19)$$

Since $G_i(s) = G(s)$, and remembering that the phasors are only determined to an arbitrary constant, we may choose $w_i = 1$, thus the corresponding phasor, $\mathbf{w}^{(n)}$, for any solution, s_n, of equation (19) is given by equations (13) as

$$\mathbf{w}^{(n)} = (1, 1, \ldots, 1)^t. \quad (20)$$

Equation (20) indicates that for this mode, all parallel channels asymptotically oscillate in-phase with the same amplitude.

The second set of solutions of equation (18) satisfies the equation

$$G(s) = 0. \quad (21)$$

This is just the characteristic equation of a single boiling channel subjected to a constant pressure drop boundary condition. The components, $w_i^{(n)}$, of the corresponding phasor, $\mathbf{w}^{(n)}$, must satisfy equation (12). That is, since $C_i(s) = C(s)$, we have for the out-of-phase components

$$\sum_{i=1}^{N} w_i^{(n)} = 0. \quad (22)$$

Equation (22) is the only restriction that the phasor $\mathbf{w}^{(n)}$ must satisfy, which means that there are $N-1$ degrees of freedom. In fact $\mathbf{w}^{(n)}$ can be thought as an 'out-of-phase' subspace of dimension $N-1$. This is similar to a semi-simple degeneracy in the classical matrix eigenvalue problem. Physically it means that any mode which satisfies the constraint given by equation (22) is possible. Thus we see that the mode of oscillation is entirely determined by the initial perturbation.

PARAMETER PERTURBATION ANALYSIS

Let us now consider a system consisting of similar parallel channels differing slightly in the value of one parameter, k (e.g. power level, inlet loss coefficient, channel diameter, etc.). It is not unreasonable to assume that the solution of this problem will be similar to the solution of a system of identical channels.

Let us define the deviations from the average value of the control parameter

$$\bar{k} = \sum_{i=1}^{N} k_i/N$$

as

$$\Delta k_i = k_i - \bar{k}. \quad (23)$$

For a given set of Δk_i, any solution of the characteristic equation, equation (15), is perturbed as

$$\Delta s = s - s^0 \quad (24)$$

where s^0 is a solution of the characteristic equation for $k = k^0 = \bar{k}$ in all channels.

To first order, the transfer functions $G_i = G(s, k_i)$ and $C_i = C(s, k_i)$ in equation (15) are given by

$$G_i = G^0 + \left.\frac{\partial G}{\partial k}\right|^0 \Delta k_i + \left.\frac{\partial G}{\partial s}\right|^0 \Delta s \quad (25a)$$

$$C_i = C^0 + \left.\frac{\partial C}{\partial k}\right|^0 \Delta k_i + \left.\frac{\partial C}{\partial s}\right|^0 \Delta s \quad (25b)$$

where superscript '0' means evaluated at $s = s^0$ and $k = k^0 = \bar{k}$.

Channel-to-channel stability

Let us now consider the 'out-of-phase' modes, corresponding to the characteristic equation $G^0 = G(s^0, k^0) = 0$ in the identical channels case. Combining equations (25) with equation (15) yields

$$\sum_{i=1}^{N}\left(C^0 + \left.\frac{\partial C}{\partial k}\right|^0 \Delta k_i + \left.\frac{\partial C}{\partial s}\right|^0 \Delta s\right) \times \prod_{j \neq i}\left(\left.\frac{\partial G}{\partial k}\right|^0 \Delta k_j + \left.\frac{\partial G}{\partial s}\right|^0 \Delta s\right) = 0. \quad (26)$$

Taking only the lowest order terms leads to

$$\sum_{i=1}^{N}\prod_{j \neq i}\left(\Delta k_j + \frac{\left.\frac{\partial G}{\partial s}\right|^0}{\left.\frac{\partial G}{\partial k}\right|^0}\Delta s\right) = 0. \quad (27)$$

The functions $\partial G/\partial k|^0$ and $\partial G/\partial s|^0$ are known provided that the solution of the identical channel system is known. Thus the only unknown is Δs. Equation (27) is a polynomial of order $(N-1)$, which gives $(N-1)$ roots, $\Delta s^{(n)}$ $(n = 1, 2, \ldots, N-1)$.

Let us define a new variable, $k^{(n)}$, such that

$$\Delta k^{(n)} = k^{(n)} - \bar{k} = -\frac{\left.\frac{\partial G}{\partial s}\right|^0}{\left.\frac{\partial G}{\partial k}\right|^0}\Delta s. \quad (28)$$

Combining equations (23), (27) and (28) we see that $k^{(n)}$ is the nth modal solution of the characteristic equation for the 'out-of-phase' modes

$$\sum_{i=1}^{N}\prod_{j \neq i}(k_j - k^{(n)}) = 0. \quad (29)$$

Equations (28) and (29) lead to $(N-1)$ solutions of the characteristic equation, equation (24). The multiple solution, s^0, which occurs in the identical channels case splits into $(N-1)$ different values, corresponding to the various values of $k^{(n)}$. Using equations (24) and (28), these solutions can be written as

$$s^{(n)} = s^0 - \frac{\left.\frac{\partial G}{\partial k}\right|^0}{\left.\frac{\partial G}{\partial s}\right|^0}(k^{(n)} - k^0). \quad (30)$$

To understand the physical meaning of equation (30), let us consider the characteristic equation of a single channel subjected to constant pressure drop boundary condition

$$G(s, k) = 0. \quad (31)$$

Equation (31) defines a curve, $s = f(k)$, in the plane s–k, passing through the point (s^0, k^0). If the parameter k changes from k^0 to $k^{(n)}$, we have, to first order

$$s = s^0 + \left.\frac{\mathrm{d}s}{\mathrm{d}k}\right|^0 (k^{(n)} - k^0). \quad (32)$$

Differentiating equation (31) yields

$$\left.\frac{\mathrm{d}s}{\mathrm{d}k}\right|^0 = -\frac{\left.\frac{\partial G}{\partial k}\right|^0}{\left.\frac{\partial G}{\partial s}\right|^0}. \quad (33)$$

Thus from equations (32) and (33)

$$s = s^0 - \frac{\left.\frac{\partial G}{\partial k}\right|^0}{\left.\frac{\partial G}{\partial s}\right|^0}(k^{(n)} - k^0). \quad (34)$$

Comparing equations (34) and (30), we see that, to first order, each $s^{(n)}$ can be thought of as being the solution of the characteristic equation

$$G(s, k^{(n)}) = 0 \quad (35)$$

where $k^{(n)}$ is given by equation (29).

Dividing equation (29) by

$$\prod_{i=1}^{N} (k_i - k^{(n)})$$

we have

$$\sum_{i=1}^{N} \frac{1}{k_i - k^{(n)}} = 0. \quad (36)$$

Figure 2 shows the left-hand side of equation (36). It can be seen that all the solutions $k^{(n)}$ of equation (29) are such that

$$k_1 \leqslant k^{(1)} \leqslant k_2 \leqslant k^{(2)} \leqslant \cdots \leqslant k_{N-1} \leqslant k^{(N-1)} \leqslant k_N \quad (37)$$

where the k_i are ordered from smaller to larger values.

Equation (35) leads to an important conclusion concerning the analysis of parallel boiling channel

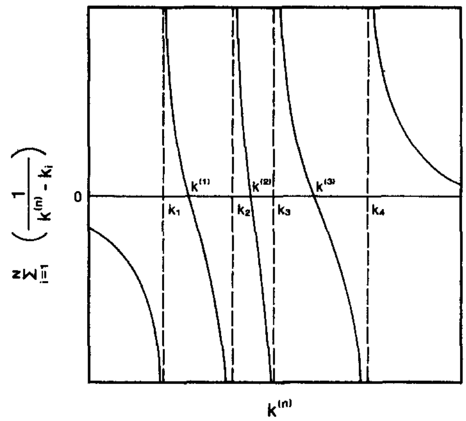

FIG. 2. An evaluation of equation (36).

instabilities. That is, to first order, the stability of each mode in coupled channel-to-channel oscillations is equivalent to the stability of an equivalent single channel subjected to a constant pressure drop. The parameter of the equivalent channel is given by equation (29). This observation is very significant since it greatly reduces the complexity of the analysis required.

Moreover, knowing how a certain parameter affects the stability of a single channel, one can predict which mode will be more unstable, and thus prevail in coupled channel-to-channel oscillations. Finally, equation (37) indicates that any channel-to-channel oscillation mode, is as stable as, or more stable than, the most unstable channel, and as unstable as, or more unstable than, the most stable one.

Another significant conclusion is that, to first order, the external loop does not influence the stability of coupled channel-to-channel oscillations. The influence of the external loop on the stability of channel-to-channel oscillations is a second-order effect and thus will only be seen for large differences between the channels.

Channel-to-channel modes of oscillation

For a system of identical channels, it has been shown that there exists an 'out-of-phase' subspace of order $(N-1)$. This subspace is defined by equation (22). When the channels are not identical, the multiple root, s^0, of the characteristic equation splits into $N-1$ different values, $s^{(n)}$. Each of these solutions determines one corresponding phasor, $\mathbf{w}^{(n)}$, in the 'out-of-phase' subspace. Moreover, due to the interaction with the loop, $\mathbf{w}^{(n)}$ may contain a small component of the corresponding phasor of the in-phase mode. Thus, to first order, the 'out-of-phase' phasor, $\mathbf{w}^{(n)}$, can be partitioned as

$$\mathbf{w}^{(n)} = \mathbf{w}^{0(n)} + \varepsilon(1, 1, \ldots, 1)^{\mathrm{T}} \quad (38)$$

where the in-phase phasor from equation (20) has been used in conjunction with ε, an unknown small parameter. The first term in equation (38) represents an out-of-phase phasor and the second term represents the in-phase component due to the loop interaction. Combining equations (25a), (38) and (13), and recalling that $G^0 = 0$, we have, to first order

$$\left(\frac{\partial G}{\partial k}\bigg|^0 \Delta k_i + \frac{\partial G}{\partial s}\bigg|^0 \Delta s\right) w_i^{0(n)} = \left(\frac{\partial G}{\partial k}\bigg|^0 \Delta k_j + \frac{\partial G}{\partial s}\bigg|^0 \Delta s\right) w_j^{0(n)} \quad (39)$$

where the products $\varepsilon\Delta s$ and $\varepsilon\Delta k_i$ are assumed to be second-order terms. Using equation (28) to eliminate Δs in equation (39) gives

$$(k_i - k^{(n)}) w_i^{0(n)} = (k_j - k^{(n)}) w_j^{0(n)}. \quad (40)$$

Remembering that $\mathbf{w}^{(n)}$ is only determined to an arbitrary constant, we may choose

$$w_1^{0(n)} = \frac{1}{k_1 - k^{(n)}}$$

such that equation (40) yields

$$w_i^{0(n)} = \frac{1}{k_i - k^{(n)}}. \quad (41)$$

It can be seen from equations (22), (36) and (41) that $\mathbf{w}^{0(n)}$ belongs to the out-of-phase subspace

$$\left(\text{i.e. } \sum_{i=1}^{N} w_i^{0(n)} = 0\right).$$

Combining equations (25b), (38) and (41) with equation (11) yields

$$\sum_{i=1}^{N} \left(C^0 + \frac{\partial C}{\partial k}\bigg|^0 \Delta k_i + \frac{\partial C}{\partial s}\bigg|^0 \Delta s\right)\left(\frac{1}{k_i - k^{(n)}} + \varepsilon\right) = 0. \quad (42)$$

Taking only first-order terms, gives

$$\sum_{i=1}^{N} \left(C^0 \varepsilon + \frac{\frac{\partial C}{\partial k}\Big|^0 \Delta k_i}{k_i - k^{(n)}} + \frac{\frac{\partial C}{\partial s}\Big|^0 \Delta s}{k_i - k^{(n)}}\right) = 0. \quad (43)$$

Remembering that $\Delta k_i = k_i - \bar{k} = k_i - k^{(n)} + k^{(n)} - \bar{k}$, equation (36) can be combined with equation (43) to give

$$\varepsilon C^0 + \frac{\partial C}{\partial k}\bigg|^0 = 0. \quad (44)$$

Thus

$$\varepsilon = -\frac{1}{C^0}\frac{\partial C}{\partial k}\bigg|^0. \quad (45)$$

The coefficient, ε, quantifies the interaction with the loop.

The nth mode of oscillation can be estimated by using equation (38) in conjunction with equations (41) and (45). That is

$$w_i^{(n)} = \frac{1}{k_i - k^{(n)}} - \frac{1}{C^0}\frac{\partial C}{\partial k}\bigg|^0. \quad (46)$$

It is interesting to note that

$$\frac{1}{C^0}\frac{\partial C}{\partial k}\bigg|^0$$

is in general a complex number and, as a consequence, a phase shift between the parallel channel oscillations may be introduced through interaction with the loop. This is in agreement with the numerical results obtained by Lahey *et al.* [11] where a noticeable phase shift was evident in the channel-to-channel oscillations interacting with an external loop.

'In-phase' mode of oscillation

Let us now consider the in-phase mode of oscillation corresponding to the characteristic equation, $C(s^0, k^0) = 0$. Combining equations (25) and (15) yields

$$\sum_{i=1}^{N} \left(\frac{\partial C}{\partial k}\bigg|^0 \Delta k_i + \frac{\partial C}{\partial s}\bigg|^0 \Delta s\right) \times \prod_{j \neq i} \left(G^0 + \frac{\partial G}{\partial k}\bigg|^0 \Delta k_j + \frac{\partial C}{\partial s}\bigg|^0 \Delta s\right) = 0. \quad (47)$$

Taking only first-order terms yields

$$\Delta s = -\frac{\frac{\partial C}{\partial k}\Big|^0}{\frac{\partial C}{\partial s}\Big|^0} \sum_{i=1}^{N} \Delta k_i = 0. \quad (48)$$

Equation (48) implies that $s = s_0$. Thus, to first order, the solution of the characteristic equation of the in-phase mode is the same as in a system of identical channels with $k = \bar{k}$. In other words, the stability of the in-phase mode of a boiling loop consisting of parallel channels with different parameters, k_i, is to first order, just the stability of the loop coupled with a single equivalent channel. The parameter k of this equivalent channel is the simple average of all channels ($\bar{k}$).

The corresponding phasor $\mathbf{w}^{(n)}$ can be calculated by combining equations (13) and (25a). Recalling that $\Delta s = 0$ from equation (48), we have

$$\left(G^0 + \frac{\partial G}{\partial k}\bigg|^0 \Delta k_i\right) w_i = \left(G^0 + \frac{\partial G}{\partial k}\bigg|^0 \Delta k_j\right) w_j. \quad (49)$$

Remembering that $\mathbf{w}_n$ is only determined to an arbitrary constant, we may choose

$$w_1 = \frac{1}{1 + \frac{1}{G^0}\frac{\partial G}{\partial k}\Big|^0 \Delta k_1}$$

such that equation (49) yields

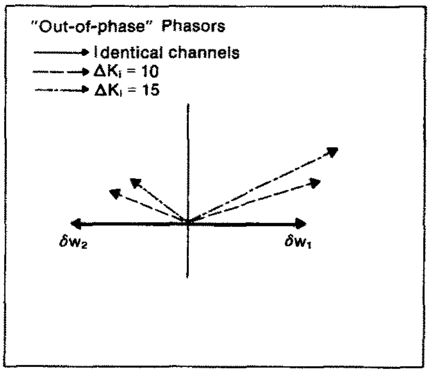

FIG. 3. Out-of-phase phasor dependence on variation in inlet loss coefficient.

$$w_i = \frac{1}{1+\frac{1}{G^0}\frac{\partial G}{\partial k}\Big|^0 \Delta k_i}. \tag{50}$$

To first order, equation (49) yields

$$w_i = 1-\frac{1}{G^0}\frac{\partial G}{\partial k}\Big|^0 \Delta k_i. \tag{51}$$

The term on the far right of equation (51) represents the relative variation, $\Delta G_i/G^0$, of the single channel transfer function, $G(k,s)$, at constant $s = s^0$, for an imposed variation, Δk_i in the parameter k from $k = \bar{k}$. It should be noted that this term is in general a complex vector, and therefore some phase shift may occur due to the interaction with the channel-to-channel oscillation modes.

APPLICATIONS

Systems of two channels

For a system of two channels, the oscillation modes are given by equations (11) and (13). The phasor corresponding to the 'in-phase' mode is given by

$$\mathbf{w}_{in} = \begin{pmatrix} G_2 \\ G_1 \end{pmatrix}. \tag{52a}$$

On the other hand, equation (51) gives a first-order estimate of the 'in-phase' phasor

$$\mathbf{w}_{in} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{G^0}\frac{\partial G}{\partial k}\Big|^0 \Delta k \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{52b}$$

where $\Delta k = k_1-\bar{k} = \bar{k}-k_2$.

The phasor corresponding to the 'out-of-phase' mode is given by

$$\mathbf{w}_{out} = \begin{pmatrix} -C_2 \\ C_1 \end{pmatrix}. \tag{53a}$$

On the other hand, the first-order estimate of the 'out-of-phase' phasor is given by equation (46), that is

$$\mathbf{w}_{out} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} + \frac{1}{C^0}\frac{\partial C}{\partial k}\Big|^0 \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{53b}$$

Figures 3 and 4 show an example of how the

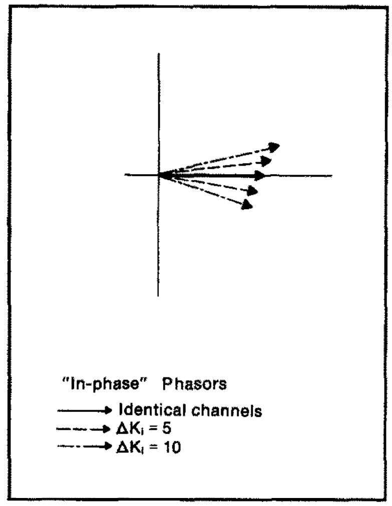

FIG. 4. In-phase phasor dependence on variation in inlet loss coefficient.

phasors $\mathbf{w}_{in}$ and $\mathbf{w}_{out}$ change as the difference between channel inlet loss coefficients increase. An asymmetry in oscillation amplitudes as well as some phase shift can be noted. A system with a downcomer and riser coupled with two channels was also considered. The transfer functions $C(s)$ and $G(s)$ were derived following the procedure shown in the Appendix. The results are given in ref. [11].

Figures 5 and 6 show an example of the variation in the phase shift between channel oscillations in the case of channels with different power skews and different inlet loss coefficients. A comparison with the exact calculation of the phase shift, equations (52a) and (53a), is also shown. It can be observed that the method presented herein constitutes a very good approximation.

System with many stable channels

Let us consider now a case with N identical stable channels in parallel with an unstable channel. In par-

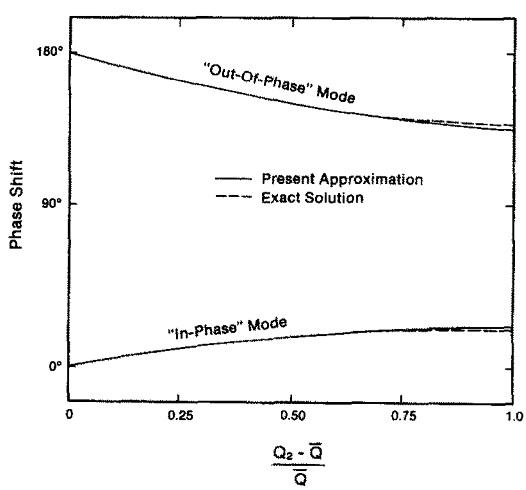

FIG. 5. Variation in phase shift of in-phase and out-of-phase modes to power skew.

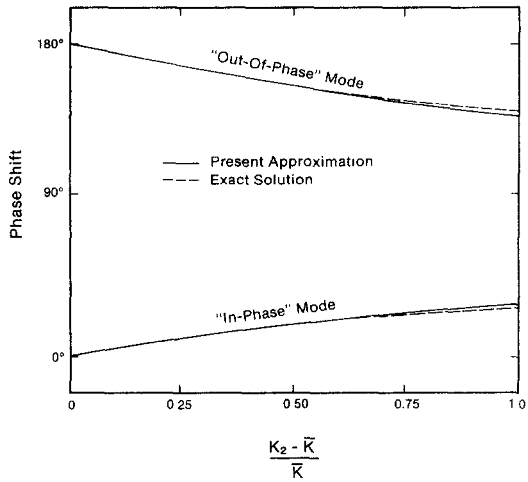

FIG. 6. Variation in phase shift of in-phase and out-of-phase modes to differences in inlet loss coefficients.

ticular, let us analyze the stability of channel-to-channel oscillations. Equation (29) gives

$$(k_{st}-k^{(n)})^{N-1}[k_{st}-(N-1)k_{un}-Nk^{(n)}] = 0. \quad (54)$$

There are $(N-1)$ solutions $k^{(n)} = k_{st}$ of equation (54) corresponding to the characteristic equation

$$G(s, k_{st}) = 0. \quad (55)$$

All the solutions of equation (55) have negative real parts since it is the characteristic equation of a single stable channel with constant pressure drop boundary condition.

The other solution of equation (54) is

$$k^{(N)} = \frac{k_{st}}{N} + \left(1 - \frac{1}{N}\right)k_{un}. \quad (56)$$

The corresponding characteristic equation of equation (56) is

$$G(k^{(N)}, s) = 0. \quad (57)$$

It can be noted that as N tends to infinity, $k^{(N)}$ tends to k_{un}, and equation (57) corresponds then to the characteristic equation of the unstable channel subjected to constant pressure drop.

CONCLUSIONS

Systems consisting of parallel boiling channels coupled with an external loop have been studied. The theory of parameter perturbation was applied to quantify the instability modes which can occur in systems with different channels. Simple analytical expressions to analyze the stability and the oscillation modes of parallel channel systems were found. Comparisons made against exact solutions show good agreement, even for large differences between channels. A notable achievement of this work is the considerable reduction in the analysis required to calculate the stability of complicated boiling systems.

Concerning the modes of oscillation, it was shown that for identical channels the system can oscillate with all channels in-phase with the external loop, or with the channels oscillating out-of-phase, while maintaining a constant flow in the loop. If the channels are different, these two modes begin to interact, and complicated modes involving coupling between the channels and the loop can be formed.

This study on channel-to-channel oscillation modes explains the disagreements arising in different experimental data. In effect, the oscillation mode is determined by the phasor $\mathbf{w}^0$ given by equation (41). However, the mode which occurs is strongly dependent on small differences between channels.

It is recommended that future developments in this field focus on performing experiments which quantify the differences between the channel's parameters. It would be interesting, for example, to obtain data from multichannel experiments to assess the limits of equation (41) as the differences between channels increase.

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APPENDIX A. TRANSFER FUNCTIONS OF A BOILING CHANNEL

Consider one of the boiling channels shown in Fig. 1. In the single-phase region the mass and energy mixture conservation equations are

$$\frac{\partial w}{\partial z} = 0 \quad \text{(A1)}$$

$$A\rho_f \frac{\partial h}{\partial t} + w\frac{\partial h}{\partial z} = q'. \quad \text{(A2)}$$

Linearizing and integrating equations (A1) and (A2) in space yields

$$\delta h_{1\phi}(z,s) = \frac{q'}{u_{0i}}\left[\frac{\exp\left(-\frac{zs}{u_{0i}}\right)-1}{s}\right]\delta u_i(s) + \exp\left(\frac{zs}{u_{0i}}\right)\delta h_i(s). \quad \text{(A3)}$$

In the linear approximation the perturbation of the boiling boundary is given by

$$\delta\lambda(s) = -\frac{q'}{w_0}\delta h(\lambda_0, s). \quad \text{(A4)}$$

In the two-phase region, the mass and energy conservation equations are

$$A\frac{\partial \rho}{\partial t} + \frac{\partial w}{\partial z} = 0 \quad \text{(A5)}$$

$$A\rho\frac{\partial h}{\partial t} + w\frac{\partial h}{\partial z} = q'. \quad \text{(A6)}$$

For a homogeneous mixture, the enthalpy and the density are related by

$$\rho = \left[v_f + \left(\frac{h-h_f}{h_{fg}}\right)v_{fg}\right]^{-1}. \quad \text{(A7)}$$

Combining equations (A5)–(A7) yields

$$\frac{\partial u}{\partial z} = \frac{q'}{A}\frac{v_{fg}}{h_{fg}} \triangleq \Omega. \quad \text{(A8)}$$

Integrating equation (A8) in space yields

$$\delta u_{2\phi}(z,s) = \delta u_i(s) - \Omega\delta\lambda(s). \quad \text{(A9)}$$

Similarly, equation (A6) can be linearized and integrated to give

$$\delta h_{2\phi}(z,s) = \left[\frac{u_0(z)}{u_{0i}}\right]^{(\Omega-s)/\Omega}\delta h(\lambda_0,s) + \left\{\frac{\left[\frac{u_0(z)}{u_{0i}}\right]^{(\Omega-s)/\Omega}-1}{s-\Omega}\right\}\frac{q'}{w_0}\delta u_{2\phi}(z,s). \quad \text{(A10)}$$

Using equation (A7) the perturbation of the two-phase density can be expressed as

$$\delta\rho(z,s) = -\rho_0^2(z)\frac{v_{fg}}{h_{fg}}\delta h(z,s). \quad \text{(A11)}$$

The total pressure drop, Δp, can be calculated by integrating the momentum conservation equation over the whole channel. That is

$$\Delta p = \int_0^L \left(\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial z} + f\rho u^2 + g\rho\right)\mathrm{d}z. \quad \text{(A12)}$$

Equation (A12) can be linearized and integrated using equations (A3), (A4), (A9), (A10) and (A11). Partitioning the different terms of the momentum equation, we obtain

$$\delta\Delta p_I = \delta\int_0^L \frac{\partial\rho u}{\partial t}\mathrm{d}z = \rho_f\lambda_0 s\delta u_i + s\rho_f I(-1)\delta u_e + \Omega\frac{v_{fg}}{h_{fg}}\rho_f^2 u_{0i}h_0' I\left(-\frac{s}{\Omega}\right)\delta\lambda - \frac{v_{fg}}{h_{fg}}\rho_f^2 u_{0i}h_0'\frac{\left[I\left(-\frac{s}{\Omega}\right)-I(-1)\right]}{s-\Omega}\delta u_e \quad \text{(A13a)}$$

$$\delta\Delta p_a = \delta\int_0^L \frac{\partial\rho u^2}{\partial z}\mathrm{d}z = -2\rho_f u_{0i}\delta u_i + 2\rho_f u_{0i}\delta u_e + u_{0e}^2\delta\rho_e \quad \text{(A13b)}$$

$$\delta\Delta p_i = \delta(K_i\rho_f u_i^2) = 2K_i\rho_f u_{0i}\delta u_i \quad \text{(A13c)}$$

$$\delta\Delta p_e = \delta(K_e\rho_e u_e^2) = 2K_e\rho_f u_{0i}\delta u_e + K_e u_{0e}^2\delta\rho_e \quad \text{(A13d)}$$

$$\delta\Delta p_f = f\delta\int_0^L \rho u^2\,\mathrm{d}z = 2f\rho_f u_{0i}\lambda_0\delta u_i + 2f\rho_f u_{0i}(L-\lambda_0)\delta u_e + f\frac{v_{fg}}{h_{fg}}\rho_f^2 u_{0i}^2 h_0'\left\{I\left(1-\frac{s}{\Omega}\right)\delta\lambda - \frac{\left[I\left(1-\frac{s}{\Omega}\right)+\lambda_0-L\right]}{s-\Omega}\delta u_e\right\} \quad \text{(A13e)}$$

$$\delta\Delta p_G = \delta\int_0^L g\rho\,\mathrm{d}z = g\frac{v_{fg}}{h_{fg}}\rho_f^2 h_0'\left\{I\left(-1-\frac{s}{\Omega}\right)\delta\lambda - \frac{\left[I\left(-1-\frac{s}{\Omega}\right)-I(2)\right]}{s-\Omega}\delta u_e\right\} \quad \text{(A13f)}$$

where

$$\delta u_e = \delta u_i - \Omega\delta\lambda \quad \text{(A14)}$$

$$\delta\rho_e = \rho_{0e}^2\frac{v_{fg}}{h_{fg}}h_0'\delta h_{2\phi}(L,s). \quad \text{(A15)}$$

The auxiliary function, $I(x)$, is defined as

$$I(x) \triangleq \int_{\lambda_0}^L \left(\frac{u_0(z)}{u_{0i}}\right)^x \mathrm{d}z = \begin{cases} \left[\frac{\left(\frac{u_{0e}}{u_{0i}}\right)^{x+1}-1}{x+1}\right]u_{0i}/\Omega & \text{if, } x \neq -1 \\ \frac{u_{0i}}{\Omega}\ln\left[1+\frac{\Omega}{u_{0i}}(L-\lambda_0)\right] & \text{if, } x = -1. \end{cases} \quad \text{(A16)}$$

UNE ANALYSE DE STABILITE ET DES MODES D'OSCILLATION DANS DES BOUCLES MULTICANAUX D'EBULLITION PAR DES METHODES DE PERTURBATION

Résumé—On étudie des systèmes de canaux parallèles d'ébullition couplés avec une boucle externe. On applique la théorie de perturbation avec opérateurs dépendant linéairement des paramètres. Cette méthode permet l'analyse des systèmes asymétriques. Des critères analytiques sont trouvés qui réduisent grandement la complexité de l'analyse des canaux parallèles couplés. En outre, il est possible par cette analyse d'expliquer plusieurs résultats apparemment contradictoires dans les expériences.

ANALYSE DER STABILITÄT UND DES SCHWINGUNGSVERHALTENS IN ANORDNUNGEN AUS PARALLELEN VERDAMPFUNGSKANÄLEN IN EINEM UMLAUFSYSTEM MITTELS STÖRUNGSVERFAHREN

Zusammenfassung—Es werden Anordnungen aus parallelen Verdampfungskanälen untersucht, die Teil eines Umlaufsystems sind. Dabei wird die Theorie der Störung von parameterabhängigen linearen Operatoren angewandt, wodurch die Untersuchung asymmetrischer Systeme ermöglicht wird. Einfache analytische Kriterien werden ermittelt, die die Komplexität der Untersuchung von gekoppelten parallelen Kanälen stark vermindern. Außerdem ist es mit dieser Untersuchungsmethode möglich, viele der scheinbar widersprüchlichen früheren Versuchsergebnisse zu erklären.

АНАЛИЗ РЕЖИМОВ УСТОЙЧИВОСТИ И КОЛЕБАНИЙ В МНОГОКАНАЛЬНЫХ ИСПАРИТЕЛЬНЫХ КОНТУРАХ МЕТОДАМИ ВОЗМУЩЕНИЯ ПАРАМЕТРОВ

Аннотация—На основе теории возмущения зависимых от параметров линейных операторов исследуется система, состоящая из параллельных испарительных каналов и внешнего контура. МетоД позволяет проводить анализ асимметричных систем. Найдены простые аналитические критерии, в значительной степени упрощающие анализ спаренных параллельных каналов. Анализ позволяет объяснить многие из ранее полученных кажущихся противоречивых экспериментальных результатов.